The Origin Of The Tilt Of Uranus' Axis

Formation and Probability Theories Examined

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Abstract

The planet Uranus is titled 97 degrees on its axis compared to its orbital plane. Formation theories of the origin of the Solar System forbid it forming this way. Ruling out a creationist viewpoint the evolutionists must explain how the planet suddenly changed its inclination and it has its first 18 moons that all orbit the same direction and nearly perfectly circular eccentricities and almost zero inclinations have changed along with the parent planet's new equator. Only three evolutionist possibilities exist, Collision, Capture or the Moon X theory.

Collision Theory

The collision theory claims that a large asteroid the size of the Earth collided with Uranus and turned its axis over 97^o. This theory is useless because if an object hit the planet and turned it over on its side the inner 18 moons would still orbit the planet on the same plane. This would mean that their orbits should now be polar rather than equatorial. Any material that was ejected into space could not have formed these moons. The angle of collision would mean that the asteroid hit the planet either on the North Pole heading towards the Sun or on the South Pole heading out of the Solar System. Debris flung into space would obviously have a polar orbit.

For this material to do a 90^o turn in space from polar to equatorial plane after bouncing of the planet's surface would require enormous torque that has no mechanism. You would need 18 precise asteroid strikes, one per moon as well as the one that hit Uranus.

Uranus Moons, 18 Precise Asteroid Strikes Probabilities

Bullseye Probability, even hitting moon

$$\Phi = \frac{\pi R^2}{\pi r^2}$$

R= Solar Systems radius, metres
r= moons radius
F = Bulls Eye Chance

Inclination probability, hitting the moon on the right location to get the new inclination

$$\Omega = \frac{360 \times 360}{I}$$

360= Longitude degrees on the moons surface, asteroid collision location
360= Latitude degrees on the moons surface, asteroid collision location
I = Moons inclination
Ω = Inclination chance

Eccentricity chance, getting the new orbital ratio right

$$E = \frac{1}{e}$$

e= Moon's Eccentricity
E = Eccentricity chance

Multiply all three probabilities by each other

$$\Sigma = \Phi \times \Omega \times E$$

Σ = All Three Probabilities

Finally find the factorial of all eighteen probabilities

$$K = \int_{18}^{1} \Sigma!$$

K = The Factorial Of All 18 Probabilities

Moons Name	Bulls Eye Chance [1/x]	Inclination Chance [1/x]	Eccentricity Chance [1/x]
Cordelia	62,499,977,273	1,528,482	3,846.15
Ophelia	56,689,321,789	1,250,965	100.81
Bianca	38,446,737,269	671,503	1,086.96
Cressida	16,023,067,399	21,600,000	2,777.78
Desdemona	24,414,053,622	1,164,944	7,692.31
Juliet	11,562,026,088	1,993,846	1,515.15
Portia	5,486,966,455	2,196,610	20,000.00
Rosalind	19,290,116,442	464,516	9,090.91
Cupid	308,641,863,075	1,296,000	769.23
Belinda	12,345,674,523	4,180,645	14,285.71
Perdita	111,111,070,707	1,296,000,000	833.33
Puck	3,810,393,371	406,015	8,333.33
Mab	159,999,941,818	970,787	400
Miranda	449,626,862	30,624	769.23
Ariel	74,702,154	498,462	833.33
Umbriel	73,176,363	632,195	10,000.00
Titania	40,210,221	381,176	909.09
Oberon	43,168,856	2,234,483	714.29

Moons Name	All Three Chances	Factorial Chances
Cordelia	3.67E+20	3.67E+20
Ophelia	7.15E+18	2.63E+39
Bianca	2.81E+19	7.37E+58
Cressida	9.61E+20	7.09E+79
Desdemona	2.19E+20	1.55E+100
Juliet	3.49E+19	5.42E+119
Portia	2.41E+20	1.31E+140
Rosalind	8.15E+19	1.06E+160
Cupid	3.08E+20	3.27E+180
Belinda	7.37E+20	2.41E+201
Perdita	1.20E+23	2.89E+224
Puck	1.29E+19	3.73E+243
Mab	6.21E+19	2.32E+263
Miranda	1.06E+16	2.46E+279
Ariel	3.10E+16	7.62E+295
Umbriel	4.63E+17	3.52E+312
Titania	1.39E+16	4.90E+328
Oberon	6.89E+16	3.37E+344

Moon X Theory

Two French astronomers [Gwenael Boue, and Jacques Laskar] propose that millions of years ago when the Solar System began, Uranus had a moon [Moon X] that was 1% the mass of Uranus. Uranus' orbit and Neptune's were close to Saturn's and Neptune was closer to the Sun than Uranus. The orbit of Uranus was inclined 17^o to the Sun's equator. [**http://arxiv.org/PS_cache/arxiv/pdf/0912/0912.0181v2.pdf**]. The Moon X orbited around Uranus and its orbit around Uranus moved from 0^o to 97^o dragging Uranus and the inner 18 moons to a different axis tilt. After 380,000 years Moon X vanished and then over a 9.62 million period Uranus and Neptune's orbit moved from the inner Solar System to their present orbits. They swapped positions as Neptune is now further from the Sun.

According to the theory Neptune's orbit once varied from 1.5 billion to 1.8 billion kilometres from the Sun. Uranus's orbit once varied from 2.01 billion to 2.56 billion kilometres from the Sun. Neptune migrated out 3 billion kilometres further from the Sun and Uranus 870 million kilometres to their current orbital radius.

Moon X mass versus other bodies

Planet	**Mass**	**Moon X Ratio**
Ganymede	15	5.7873
Titan	13.452	6.4533
Mercury	33.03	2.6282
Mars	64.21	1.352
Moon X	**86.81**	**1**
Venus	486.9	0.1783
Earth	597.6	0.1453

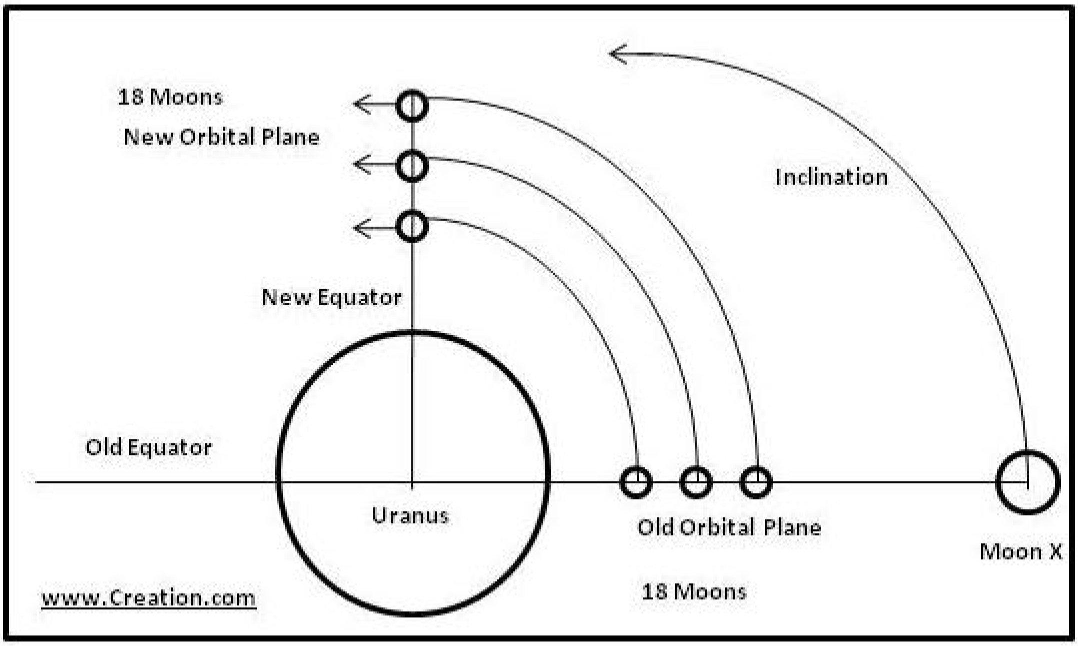
All these assumptions are of course unprovable. Moon X is bigger than Mars and there is no evidence it ever existed. There is no evidence Neptune ever orbited closer to the Sun than Uranus or their orbits swapped. There is no evidence that Uranus orbit was ever inclined 17^o to the Sun's equator.

$$I = 2\pi R \times \frac{17}{360}$$

Hypothetical original inclination = 742 million kilometres. The actual inclination of Uranus now equals 39 million kilometres. A difference of 703,264,112 kilometres!

How long would Moon X take to turn all 18 moons from 0 to 97 degrees?

0^o to 97^o



Average Inclination Time

This is half way between maximum and minimum time. The moons have to move 97^o which is just over a quarter circle = $2\pi r$ x (97/360). I tried two different methods to determine the average gravitational force over time that Moon X would exert on the other moons. The first method is to calculate the distance between Moon X and Moon D using the distance between them when Moon D is at the four points of the compass, N, S, E, W [0^o, 90^o, 180^o 270^o]

T= Inclination Period, Seconds
R= Moon X, Orbital Radius, metres
M = Mass of Moon X, kilograms
r = Moon D orbital radius, metres
G = Gravitational constant
U = Radius of Uranus
p = Radius of Moon D
x = number of Uranus radii
A = Average distance over time

$$N = (x+2)U + r + p + R$$

$$S = (R+r) - (xU + p)$$

$$E = \sqrt{(U+R+r)^2 + [(x+1)U + p)]^2}$$

$$W = \sqrt{(U+R+r)^2 + [(x+1)U + p]^2}$$

North.

If the two moons are on opposite sides of the planet the diameter of Uranus has to be added to x. The orbital radius of each moon must be added together.

South
When they are at their closest the orbital distances must be subtracted.

East and West
They can be determined by using Pythagoras theorem of the sides of right angle triangle.

$$A = \frac{N+S+E+W}{4}$$

$$A = \frac{N+S+E+W}{4} = \frac{5.21768\times10^{6}}{4} = 1.3044\times10^{6}\ Kilometres$$

$$T = [2\pi r \times \frac{97}{360}] \div [\frac{GM}{A^2}]$$

Average Inclination Velocity [0 ^{o} to 97^{o}]
V = Average Inclination Velocity, Metres/Second
M = Mass of Moon X, kg's
A = Average distance over time
G = Gravitational constant

$$V_{AVG} = \frac{GM}{A^2}$$

Minimum Inclination Velocity
Both moons are on opposite sides of the planet so their orbital distances must be added. Metres/second

$$V_{MIN} = \frac{GM}{(R+r)^2}$$

Maximum Inclination Velocity
Both moons are on same side of the planet so their orbital distances must be subtracted.

$$V_{MAX} = \frac{GM}{(R-r)^2}$$

360 Degrees Method
We use Microsoft Visual Basic and Excel and calculate the distance between Moon X [X, Y] and Moon D when Moon D [x, y] is at all 360 degrees. We add up all the distances and divide by 360.

U = Uranus radius, metres
R = Moon D, orbital radius
x = Moon D, x coordinate

y = Moon D, y coordinate
θ = Angle 0 to 360
Y = Average Distance
β = Distance from Moon X to Moon D

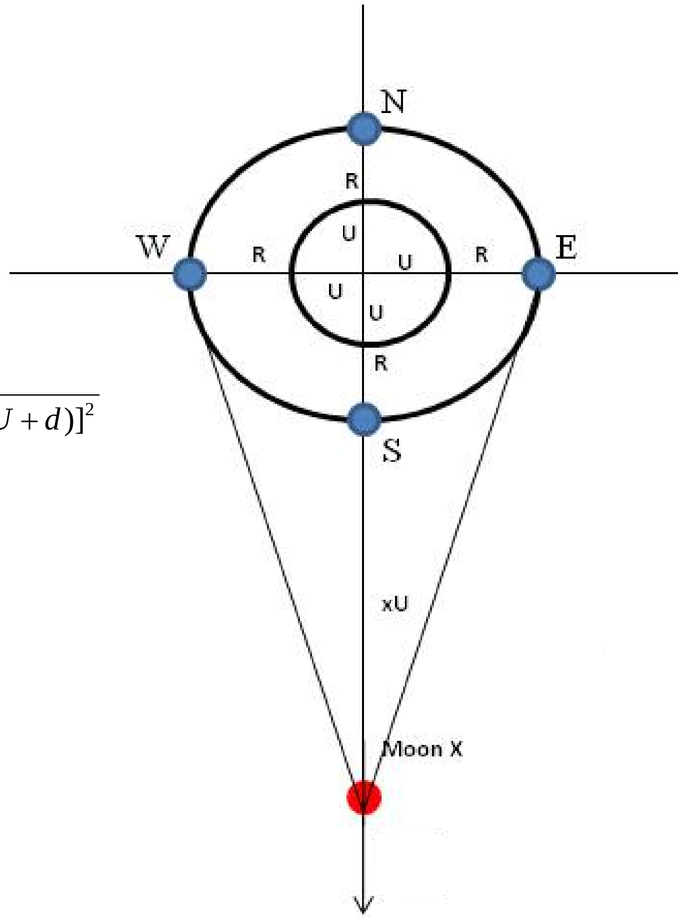
$$x = R \times Cos(\theta)$$

$$y = R \times Sin(\theta)$$

$$Y = xU + d$$

$$X = 0$$

$$\beta = \sqrt{(x - X)^2 + (y - Y)^2}$$



$$\beta = \sqrt{[(R \times Cos(\theta)) - 0]^2 + [(R \times Sin(\theta)) - (xU + d)]^2}$$

$$\psi = \frac{\sum_{360}^{0} \beta}{360}$$

$$V_{AVG} = \frac{GM}{\psi^2}$$

$$T = [2\pi r \times \frac{97}{360}] \div [\frac{GM}{\psi^2}]$$

Average Distances Between Both Moons using both Methods
Moon X orbital radius = 50 Uranus radii

Moon's	360 Degrees	Compass	Percentage	Years
Name	Method	Method	Agreement	Difference
Cordelia	78,815	78,857	99.9468	42
Ophelia	85,186	85,234	99.9439	48
Bianca	93,764	93,820	99.9401	56
Cressida	97,897	97,958	99.9383	60
Desdemona	99,315	99,377	99.9377	62
Juliet	102,020	102,085	99.9365	65
Portia	104,782	104,850	99.9352	68
Rosalind	110,874	110,949	99.9326	75
Cupid	118,629	118,713	99.9292	84
Belinda	119,354	119,439	99.9289	85
Perdita	121,209	121,296	99.9281	87
Puck	136,483	136,590	99.9214	107
Mab	155,209	155,343	99.9135	134
Miranda	205,967	206,189	99.8924	222
Ariel	305,991	306,431	99.8564	440
Umbriel	431,142	431,882	99.8287	740
Titania	731,565	732,302	99.8993	737
Oberon	1,023,805	1,021,424	100.2331	2,381

Inclination times.
Moon X orbital radius = 50 Uranus Radii

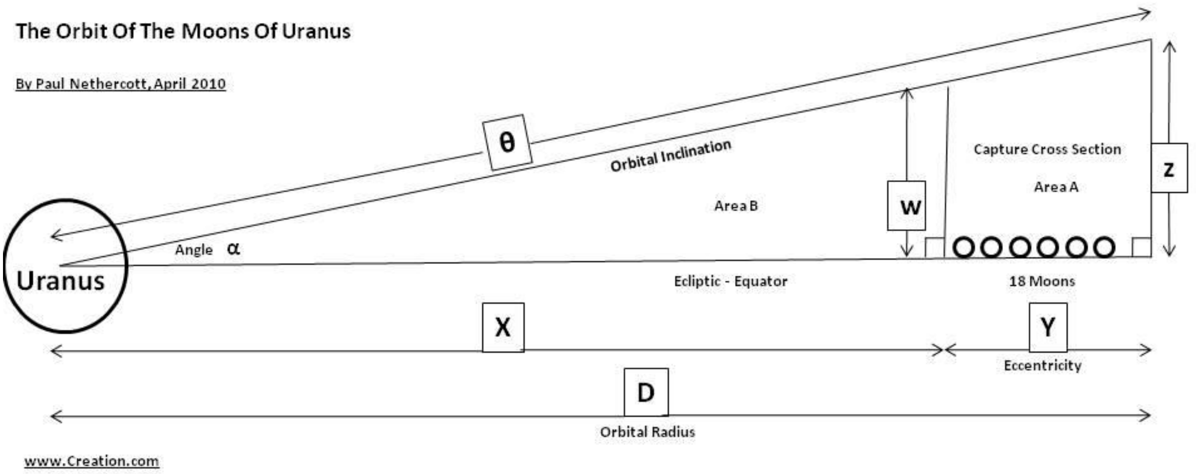
Moon's	Mass = 0.1	Mass = 0.01	Mass = 0.001	Mass = 0.005
Name	Years	Years	Years	Years
Cordelia	7,882	78,815	788,153	157,631
Ophelia	8,519	85,186	851,858	170,372
Bianca	9,376	93,764	937,640	187,528
Cressida	9,790	97,897	978,971	195,794
Desdemona	9,931	99,315	993,148	198,630
Juliet	10,202	102,020	1,020,204	204,041
Portia	10,478	104,782	1,047,823	209,565
Rosalind	11,087	110,874	1,108,739	221,748
Cupid	11,863	118,629	1,186,292	237,258
Belinda	11,935	119,354	1,193,539	238,708
Perdita	12,121	121,209	1,212,087	242,417
Puck	13,648	136,483	1,364,826	272,965
Mab	15,521	155,209	1,552,087	310,417
Miranda	20,597	205,967	2,059,674	411,935
Ariel	30,599	305,991	3,059,910	611,982
Umbriel	43,114	431,142	4,311,421	862,284
Titania	73,156	731,565	7,315,647	1,463,129
Oberon	102,380	1,023,805	10,238,047	2,047,609

If Moon X mass equals 0.01 of Uranus' mass, at a distance of 50 Uranus radii it would take over one million years to tilt all 18 moons 97 degrees. To get within the 380,000 year time frame that Laskar proposes, Moon X would have to orbiting twice as close at a distance of 26 Uranus radii. This would mean that its gravitational force on Uranus would be four times stronger and would alter vastly the tilting effect on the planet.

You need 380,000 years to get to 97 degrees and the first attempt failed by 643,805 years. The excess time equals 2.69 times the minimum time required. Multiply the Moon X mass by double the gravity for 26 radii and the extra gravity would have tilted Uranus 194 degrees [2 x 97] in the 380,000 years. According to the Laskar theory, at the end of the 380,000 years Uranus had a close encounter with Saturn which the gravity ejected the Moon X before it could tilt the planet any more. While Laskar's theory tries to answer how Uranus tilted on its axis it does not answer how the moons all got tilted exactly.

Capture Theory

What are the time limits and probability limits of 18 moons being captured in such precise orbits? The orbital eccentricities vary from 99.008% to 99.995% perfectly circular orbits. Inclinations vary from 0.013% to 0.0000003%.



For a moon to get captured it must enter the cross section capture area. What is the surface area of the capture cross section for capturing moons?

Length of line W, Triangle θXW

$$W = (D - Y) \times \tan(\alpha\left[\frac{\pi}{180}\right])$$

Area B, Triangle θXW

$$B = \tfrac{1}{2} \times (D - Y) \times W$$

$$B = \frac{1}{2} \times (D - Y)^2 \times \tan(\alpha[\frac{\pi}{180}])$$

Length of line Z, Triangle θDZ

$$Z = D \times \tan(\alpha[\frac{\pi}{180}])$$

Area of Triangle θDZ

$$Area = \frac{1}{2}[D \times Z]$$

Area A, Capture Cross Section, Big triangle area minus small triangle area.

$$A = [\frac{1}{2} D^2 \times \tan(\alpha[\frac{\pi}{180}])] - [\frac{1}{2}(D - Y)^2 \times \tan(\alpha[\frac{\pi}{180}])]$$

D = Orbital radius, centre of both objects, metres
α = angle of inclination
Y = Eccentricity, metres
A = Surface area, metres

How much time for searching the Solar Systems volume?

R = Radius of Solar System, metres
T = Search time, Seconds
Φ = Nebula Disk thickness, metres
A = Area A, square metres
V = planets orbital velocity, metres/second

$$T = \frac{\pi R^2 \Phi}{A \times V}$$

P = Capture Probability In 5 billion years [1.6 x 10^{17} seconds]

$$P = \frac{\pi R^2 \Phi}{A \times V \times 1.6 \times 10^{17}}$$

A = Area A, square metres
V = planets orbital velocity, metres/second
R = Radius of Solar System, metres
Φ = Nebula Disk thickness, metres

Probability of capturing 18 moons in 5 billion years

$$\int_{18}^{1} P!$$

Answer = 10^{-528}

How many spare moons needed for capture to be probable?
M = Mass of spare moons needed
Φ = Nebula Disk thickness
m = Mass of captured moon
A = Area A, square metres
V = planets orbital velocity, metres/second

$$M = \frac{\pi R^2 \Phi\ m}{A \times V \times 1.6 \times 10^{17}}$$

Moon's Name	5 Billion Year Chance [1/x]	1/x
Cordelia	334,922,307,547,352,000,000,000,000,000,000	3.35E+32
Ophelia	112,999,843,433,572,000,000,000,000,000	1.13E+29
Bianca	2,581,278,701,271,220,000,000,000,000,000	2.58E+30
Cressida	14,685,289,756,760,000,000,000,000,000,000,000	1.47E+34
Desdemona	309,310,016,404,905,000,000,000,000,000,000	3.09E+32
Juliet	31,579,760,711,805,100,000,000,000,000,000	3.16E+31
Portia	6,003,667,084,545,280,000,000,000,000,000,000	6.00E+33
Rosalind	44,280,008,940,382,200,000,000,000,000,000	4.43E+31
Cupid	1,884,924,481,817,200,000,000,000,000,000	1.88E+30
Belinda	6,602,781,732,329,460,000,000,000,000,000,000	6.60E+33
Perdita	2,030,473,418,629,890,000,000,000,000,000,000,000	2.03E+36
Puck	12,422,702,389,405,700,000,000,000,000,000	1.24E+31
Mab	98,120,808,332,714,600,000,000,000,000	9.81E+28
Miranda	117,118,149,030,183,000,000,000,000	1.17E+26
Ariel	7,694,092,729,023,080,000,000,000,000	7.69E+27
Umbriel	471,836,829,122,905,000,000,000,000,000	4.72E+29
Titania	197,445,480,622,722,000,000,000,000	1.97E+26
Oberon	1,304,525,565,526,590,000,000,000,000	1.30E+27

Probability of 18 moons randomly achieving almost perfectly circular, non inclined, clockwise orbits
Ψ = Total orbital configuration probability
Ψ = eccentricity × inclination × clockwise
18 Moons. Multiple all 18 chances for each moon by each other.

$$\psi = \int_{18}^{1} [\frac{e}{1}]! \times \int_{18}^{1} [\frac{i}{180}]! \times \frac{1}{2^{18}}$$

Answer = 4.569 x 10^{-124}

Clockwise/anti Clockwise Probability
What is the probability of capturing 18 moons in clockwise versus anti clockwise orbits?
Multiply all 18 probabilities

$$P_c = [\frac{1}{2^{18}}]$$

Answer = 1/262,144

Eccentricity Probability
Multiply all 18 probabilities for each moon

$$P_e = \int_{18}^{1} [\frac{e}{1}]!$$

Answer = 3.15 x 10^{-59}

Inclination Probability
Multiply all 18 probabilities for each moon

$$P_i = \int_{18}^{1} [\frac{i}{180}]!$$

Answer = 5.3 x 10^{-59}

T = Moon X Orbital Period, Seconds
R = Orbital Radius, metres
M = Mass of planet, kg's
m = Mass of Moon, kg's
G = Gravitational constant
T = 3,882,476 seconds

$$T = \sqrt{\frac{4\pi^2 R^3}{G \times (M + m)}}$$

Moon's Orbital Kinetic Energy
[Units, Sun's Energy output per second]

$$E = \frac{1}{2} MV^2 \div 3.846 \times 10^{26}$$

Energy = 50,183 L_{Θ}

V=Moon's Orbital Velocity, metres/second

$$V = \sqrt{\frac{4\pi^2 R^3}{G \times (M + m)}} \div 2\pi R$$

Velocity = 2108.703 metres/second

A Close Encounter With Saturn

Introduction

According to Laskar the reason Moon X no longer exists is that it had a close encounter with Saturn which ejected it into outer space. If Moon X did have an orbit close enough [25 Uranus radii] to Uranus to tilt all the moons of Uranus, how close would Saturn have to get to Uranus to eject Moon X? What would be the effects on all the other moons?

The gravitational attraction between Moon X and Uranus is thus

Moon X mass, 8.69 x 10^{23} kilograms
Uranus mass, 8.69 x 10^{25} kilograms
Orbital radius, 650,556,000 metres
Force= 1.09002 x 10^{22} Newtons

Gravitational Force

F=Newtons
G=6.673 x 10^{-11}
U= Uranus mass, kilograms
S= Saturn's mass, kilograms
x= Moon X mass, kilograms
R= orbital radius, metres

$$F = \frac{GUx}{R^2}$$

For Saturn's gravitional force on Moon X to equal that of Uranus the distance between them must be defined thus

$$D = \sqrt{\frac{GSx}{(GUx) \div R^2}}$$

The mass of Saturn = 5.68 x 10^{26} kilograms

D= 1,734,848,110 metres

The two planets are orbiting the same distance form the Sun. Since Saturn's orbital velocity is 9.609 kilometres per second, their relative speed is 20 kilometres per second. The spreadsheet and Visual Basic macros determine that for Moon X to reach escape velocity the minimum distance of approach = 765,000,000 metres.

$$V = \sqrt{\frac{2GM}{R + r}}$$

V = Escape velocity, metres/second
G=6.673 x 10^{-11}
r=Orbital radius
R= Uranus Radius

The escape velocity for Moon X = 4,140 metres/second. The planet Saturn approaches Uranus at a distance of 1,734,848,110 metres. Saturn's gravity begins to overpower Uranus influence on Moon X. After reaching a distance of 765,000,000 metres the two begin to move apart.

The encounter will eject Moon X! It will also destroy all the good Moon X has done.

Below is a table showing how many kilometres the moons will be inclined compare to what the actual values are today.

Uranus Moons Name	Encounter Inclination Kilometres	Actual Inclination, Kilometres
Cordelia	87,962	74
Ophelia	176,425	97
Bianca	265,562	199
Cressida	355,026	6.5
Desdemona	444,602	122
Juliet	534,395	73
Portia	624,408	68
Rosalind	714,914	341
Cupid	806,044	131
Belinda	897,227	41
Perdita	988,562	0.13
Puck	1,081,198	479
Mab	1,175,394	228
Miranda	1,274,058	9,561
Ariel	1,382,309	868
Umbriel	1,504,532	953
Titania	1,669,925	2,588

The degrees of inclination caused by the encounter. The ratio of the encounter values over the real today values shows the encounter would put some inclinations out of place by a factor of up to 740 thousand

Uranus Moons Name	After Encounter, Degrees	Today's Inclination, Degrees
Cordelia	6.59	0.08479
Ophelia	13.21	0.1036
Bianca	19.88	0.193
Cressida	26.58	0.006
Desdemona	33.29	0.11125
Juliet	40.01	0.065
Portia	46.75	0.059
Rosalind	53.52	0.279
Cupid	60.35	0.1
Belinda	67.17	0.031
Perdita	74.01	0.0001
Puck	80.95	0.3192
Mab	88	0.1335
Miranda	95.38	4.232
Ariel	103.49	0.26
Umbriel	112.64	0.205
Titania	125.02	0.34

In the spreadsheet rows 34 to 64 are the encounter with the moons between Uranus and Saturn so both planets are pulling them in opposite directions. The second grid [rows 67 to 95] is the moons [except Moon X] on the opposite side of Uranus so both the planets are pulling the moons in the same direction.

The encounter can significantly damage any good Moon X can do and change all the inclinations of the following moons into the wrong angle of inclination:

Bianca	Cressida	Desdemona	Juliet
Portia	Rosalind	Cupid	Belinda
Perdita	Puck	Mab	Ariel
Umbriel	Titania	Oberon	

Cordelia, Ophelia and Miranda would be the only moons either not affected or moved into the right plane.

The Tilt of Pluto

Pluto is tilted 120 degrees and has three moons [Charon, Nix and Hydra] that all orbit on the equatorial plane.
http://en.wikipedia.org/wiki/Pluto.

Moon	Inclination	Eccentricity
Charon	**0.001**	**0.0022**
Nix	**0.195**	**0.003**
Hydra	**0.212**	**0.0051**

The probability of the same scenario [another Moon X] happening with Pluto as well is zero. What is the probability of these moons being captured in such a precise manner?

Inclination [1/x]	Eccentric [1/x]	Total Both [1/x]	Clockwise	All Probabilites
180,000	**455**	**81,818,182**	**2**	**163,636,364**
166,153,846	**151,515**	**25,174,825,174,825**	**4**	**100,699,300,699,301**
141,074,020,319	**29,708,853**	**4,191,147,365,398,200,000**	**8**	**33,529,178,923,185,600,000**

The inclination probability = all three multiplied by each other.

$$P_i = \frac{0.001}{180} \times \frac{0.195}{180} \times \frac{0.212}{180} = \frac{1}{141{,}074{,}020{,}319}$$

$$P_i = \int_3^1 [\frac{i}{180}]!$$

The eccentricity probability = all three multiplied by each other.

$$P_e = \frac{0.0022}{1} \times \frac{0.0.003}{1} \times \frac{0.00051}{1} = \frac{1}{29,708,853}$$

$$P_e = \int_3^1 [\frac{e}{1}]!$$

The clockwise probability = all three multiplied by each other.

$$P_c = [\frac{1}{2^3}] = 8$$

All three probabilities multiplied =

$$P = \frac{1}{33,529,178,923,185,600,000}$$

What is the cross section area [square kilometres] that the moon must be captured in?

Inclination, km's	Eccentricity, Km's	Cross Section Area
0.31	38.58	5.91
165.84	146.12	12,116.59
239.67	330.22	39,572.76

$$A = [\frac{1}{2} D^2 \times \tan(\alpha [\frac{\pi}{180}])] - [\frac{1}{2}(D - Y)^2 \times \tan(\alpha [\frac{\pi}{180}])]$$

Area Chance [1/x]	Capture - Years	5 Billion Year Chance [1/x]
3.45E+38	1.09E+46	2,178,815,343,528,210,000,000,000,000,000,000,000
8.20E+31	2.59E+39	517,692,240,974,860,000,000,000,000,000,000
7.69E+30	2.43E+38	48,533,278,657,085,900,000,000,000,000,000

The area chance equals the probability that that imaginary cross section net will go through the centre of a lonely moon. To both be in the same place at the same time the probabilities must be squared.

$$P = \frac{\pi R^2 \Phi}{A \times V \times 1.6 \times 10^{17}}$$

A = Area A, square metres
V = planets orbital velocity, metres/second
R = Radius of Solar System, metres

Φ = Nebula Disk thickness, metres

Spare Material Needed For Moons To Be Captured

Moon's Mass (× 10^{15} kg)	Capture Mass (Solar Mass)	5 Billion Year Chance [1/x]
1,520,000	1.67E+30	2.18E+36
2,000	5.21E+20	5.18E+29
2,000	4.88E+19	4.85E+28

Orbits Worksheet

In this worksheet you have different masses for Moon X. Mass is a fraction of the total mass of Uranus, 0.1, 0.01, 0.001, 0.0001, 0.0005 respectively. These are the values used by Laskar. If you look at my spreadsheet you will see that the Moon X with mass 0.01 could not tip the other moons 97 degrees in 380,000 years unless its orbital distance was less than 26 Uranus radii from the planet's surface.

Laskar theory allows only 15 metres variation in his theories:

"For each of the 17 planet migrations, we performed 100 integrations varying the initial semi-major axis of the satellite by a small amount (15 meters). The final obliquity distribution is given in figure 3. In 644 cases, the obliquity does not exceed 10 degrees because the satellite is ejected at the first encounter before the increase of the inclinations." Page 2, end paragraph.

Changing the orbital distance of Moon X from the surface of the planet will radically affect its inclination force on the planet's axis.

Conclusion

The formation of Uranus is only possible by instantaneous creation by God. Its ring system would have been destroyed by a close encounter with Saturn and would not have the zero inclination for all rings that it has today.